Homework 1 (Updated Jan. 29)
Due: Wednesday, February 5
MAT 308, Spring 2025

Problem 1. Solve the following initial value problems. In each case the solution should be a function $\mathbb{R} \to \mathbb{R}$.

(a) $\mathrm{d}^2 s/\mathrm{d}t^2 = \sin t;\ s(0) = 1, \mathrm{d}s/\mathrm{d}t(0) = 1$

(b) $y' = xe^x;\ y(0) = 1$

(c) $\mathrm{d}u/\mathrm{d}v = v(1 - v);\ u(0) = 1$

(d) $\mathrm{d}^3 z/\mathrm{d}t^3 = 1;\ z(0) = \mathrm{d}z/\mathrm{d}t(0) = \mathrm{d}^2 z/\mathrm{d}t^2(0) = 0$

(e) $\mathrm{d}x/\mathrm{d}t = \arctan t;\ x(0) = 0$

Problem 2. A projectile is fired directly upward from ground level at a velocity of 1000 meters per second. Assume there is no wind resistance, and that the acceleration due to gravity is 10 meters per second per second.

(a) Formulate an initial value problem that determines the height of the projectile as a function of time.

(b) What is the maximum altitude attained by the projectile, and at what time does it get there?

Problem 3. Find a solution to each of the following initial value problems. In each case the solution should be a function $\mathbb{R} \to \mathbb{R}$.

(a) $y' = 2xy,\ y(0) = 2$

(b) $\mathrm{d}z/\mathrm{d}t = t/z^2,\ z(1) = 0$

Problem 4. Suppose that a 400 liter tank contains 10 kilograms of salt in solution at time $t = 0$, and that pure water is being added to it at a rate of 2 liters per minute, and the resulting mixture (which we assume is always fully homogeneous) is being drawn off at a rate of 2 liters per minute.

(a) Find an initial value problem describing the amount of salt $s(t)$ in the tank as a function of time.

(b) Is the amount of salt in the tank increasing or decreasing?

(c) How much salt will remain in the tank after a very long time – i.e., what is $\lim_{t\to\infty} s(t)$?

(d) Suppose that the 2 liters of water being added every minute contains 0.05 kilogram of salt, and that, instead of drawing off 2 liters of water every minute, we draw off 1 liter of water per minute, and boil away 1 liter of water as steam.

Answer parts (a)-(c) in this situation.

Update Jan. 28: $\arctan x$ changed to $\arctan t$ in 1 (e).
Update Jan. 29: $v(0)$ changed to $u(0)$ in 1 (c).