Homework 7

Due: Wednesday, March 6

Math 435, Fall 2024

Problem 1. Prove $\mathrm{d}(\omega \wedge \eta) = \mathrm{d}\omega \wedge \eta + (-1)^k \omega \wedge \mathrm{d}\eta$ in the following cases: (i) ω is a 0-form, (ii) $\omega = \mathrm{d}x^{i_1} \wedge \cdots \wedge \mathrm{d}x^{i_k}$ and $\eta = \mathrm{d}x^{j_1} \wedge \cdots \wedge \mathrm{d}x^{j_l}$, (iii) the general case.

Problem 2. For any smooth function $f \colon \mathbb{R}^n \to \mathbb{R}$, find a differential form $\alpha \in \Omega^{n-1}(\mathbb{R}^n)$ such that $\mathrm{d}\alpha = f\,\mathrm{d}x^1 \wedge \cdots \wedge \mathrm{d}x^n$.

Problem 3. Spivak 5-1

Problems. **5-1. If M is a k-dimensional manifold-with-boundary, prove that ∂M is a $(k-1)$-dimensional manifold and $M - \partial M$ is a k-dimensional manifold.**

Problem 4. Spivak 5-5

5-5. Prove that a k-dimensional (vector) subspace of $\mathbf{R}^n$ is a k-dimensional manifold.

Problem 5. If $f \colon \mathbb{R}^n \to \mathbb{R}^m$, the **graph** of f is $\{(x, y) \in \mathbb{R}^n \times \mathbb{R}^m \mid y = f(x)\}$. Show that if f is smooth, then the graph of f is an n-dimensional manifold.